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5–10 Factoring by Grouping

Objective: To factor a polynomial by grouping terms.

Example 1 Factor:
a. $3(x - y) + w(x - y)$
b. $m(m + 3n) - (m + 3n)$
c. $r(p - q) + s(p - q) + t(p - q)$

Solution Use the distributive property: $ba + ca = (b + c)a$.

This property is valid when a represents any polynomial. For example:

If $a = x - y$, you have $b(x - y) + c(x - y) = (b + c)(x - y)$.

a. $3(x - y) + w(x - y) = (3 + w)(x - y)$

b. $m(m + 3n) - (m + 3n) = m(m + 3n) - 1(m + 3n)$
$= (m - 1)(m + 3n)$

c. $r(p - q) + s(p - q) + t(p - q) = (r + s + t)(p - q)$

Factor.

1. $2(x + y) + z(x + y)$ **(2 + z)(x + y)**
2. $5(a - b) + c(a - b)$ **(5 + c)(a - b)**
3. $e(f + g) - 4(f + g)$ **(e - 4)(f + g)**
4. $w(x - y) - 6(x - y)$ **(w - 6)(x - y)**
5. $(c + 2d) - e(c + 2d)$ **(1 - e)(c + 2d)**
6. $2c(a - b) - (a - b)$ **(2c - 1)(a - b)**
7. $2x(m - n) - (m - n)$ **(2x - 1)(m - n)**
8. $r(p - q) - (p - q)$ **(r - 1)(p - q)**
9. $3u(u - 2v) + v(u - 2v) + (u - 2v)$ **(3u + v + 1)(u - 2v)**
10. $c(a + b) - d(a + b) + e(a + b)$ **(c - d + e)(a + b)**

Example 2 Factor $7(a - 2) - a(2 - a)$.

Solution Notice that $a - 2$ and $2 - a$ are opposites.

$7(a - 2) - a(2 - a) = 7(a - 2) - a[-(a - 2)]$ Write $-(a - 2)$ for $2 - a$.
$= 7(a - 2) + a(a - 2)$ Use the distributive property.
$= (7 + a)(a - 2)$

Check: $(7 + a)(a - 2) = 7a - 14 + a^2 - 2a$
$= (7a - 14) + (a^2 - 2a)$
$= 7(a - 2) - (2a - a^2)$
$= 7(a - 2) - a(2 - a)$ √

Therefore, $7(a - 2) - a(2 - a) = (7 + a)(a - 2)$.

Factor. Check by multiplying the factors.

11. $2x(m - n) - (n - m)$ **(2x + 1)(m - n)**
12. $w(x - y) - 7(y - x)$ **(w + 7)(x - y)**
13. $6(r - s) + t(s - r)$ **(6 - t)(r - s)**
14. $6(m - n) + p(n - m)$ **(6 - p)(m - n)**
15. $u(v - 3) + 3(3 - v)$ **(u - 3)(v - 3)**
16. $3x(x - y) + y(y - x)$ **(3x - y)(x - y)**
17. $x(x - 5) - (5 - x)$ **(x + 1)(x - 5)**
18. $h(h - 6) - 2(6 - h)$ **(h + 2)(h - 6)**

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5–10 Factoring by Grouping *(continued)*

Example 3 Factor $ax - 2x + ay - 2y$.

Solution 1
$ax - 2x + ay - 2y = (ax - 2x) + (ay - 2y)$ Group terms with common factors.
$= x(a - 2) + y(a - 2)$ Factor each group of terms.
$= (x + y)(a - 2)$ Use the distributive property.

Solution 2
$ax - 2x + ay - 2y = (ax + ay) - (2x + 2y)$ Group terms with common factors.
$= a(x + y) - 2(x + y)$ Factor each group of terms.
$= (a - 2)(x + y)$ Use the distributive property.

Factor. Check by multiplying the factors.

19. $2a + ab + 2c + bc$ **(a + c)(2 + b)**
20. $rs - 6r + st - 6t$ **(r + t)(s - 6)**
21. $x^2 - 3x + xy - 3y$ **(x + y)(x - 3)**
22. $u^2 + 3u + uv + 3v$ **(u + 3)(u + v)**
23. $xy - xz - 3y + 3z$ **(x - 3)(y - z)**
24. $5t - 10 - st + 2s$ **(5 - s)(t - 2)**
25. $mx + m + 3x + 3$ **(x + 1)(m + 3)**
26. $5x - 5y + wx - wy$ **(5 + w)(x - y)**
27. $5m^3 - 3m^2 + 10m - 6$ **$(m^2 + 2)(5m - 3)$**
28. $2a^3 + a^2 - 6a - 3$ **$(a^2 - 3)(2a + 1)$**
29. $a^2 - 3ab + ac - 3bc$ **(a + c)(a - 3b)**
30. $2ab - b - 4a + 2$ **(b - 2)(2a - 1)**
31. $2u^3 - u^2 - 4u + 2$ **$(u^2 - 2)(2u - 1)$**
32. $x^3 - 4x^2 - x + 4$ **$(x^2 - 1)(x - 4)$** $(x+1)(x-1)(x-4)$

Example 4 Factor $(a + 2b)^2 - c^2$ as a difference of two squares.

Solution $(a + 2b)^2 - c^2 = [(a + 2b) + c][(a + 2b) - c]$ Use the pattern $a^2 - b^2 = (a + b)(a - b)$.
$= (a + 2b + c)(a + 2b - c)$

Factor as a difference of squares.

33. $(a - b)^2 - 4c^2$ **(a - b + 2c)(a - b - 2c)**
34. $(x + 3y)^2 - 16z^2$ **(x + 3y + 4z)(x + 3y - 4z)**
35. $x^2 - (y + z)^2$ **(x + y + z)(x - y - z)**
36. $9p^2 - (q - 2r)^2$ **(3p + q - 2r)(3p - q + 2r)**
37. $m^2 - (n + 3)^2$ **(m + n + 3)(m - n - 3)**
38. $h^2 - (k - 6)^2$ **(h + k - 6)(h - k + 6)**
39. $m^2 - (n - 1)^2$ **(m + n - 1)(m - n + 1)**
40. $4(x - y)^2 - 25$ **(2x - 2y + 5)(2x - 2y - 5)**

Mixed Review Exercises

Solve.

1. $-10 + x = -27$ **{-17}**
2. $-n + 8 = 3$ **{5}**
3. $16 + x = 34$ **{18}**
4. $13 = 1 + 3x$ **{4}**
5. $9m - 6m = 27$ **{9}**
6. $4n - 2n + 6 = 12$ **{3}**
7. $12x = 600$ **{50}**
8. $-11m = 143$ **{-13}**
9. $7b = 105$ **{15}**
10. $9n = 3n - 30$ **{-5}**
11. $17m = 44 + 13m$ **{11}**
12. $9y + 3 = 3(17 - y)$ **{4}**